STUDENT NAME____

CLASS DAYS/TIME____

MATH 102, COLLEGE ALGEBRA

UNIT 3 LECTURE NOTES

JILL TRIMBLE, BLACK HILLS STATE UNIVERSITY

Math102 College Algebra **Unit 3 Outcome/Homework 1**

Students will be able to add, subtract, multiply, divide, and simplify rational expressions and solve rational equations on examinations, quizzes, and homework problems. (E-text Section P.6, 1.2)

Rational Expression

$$\frac{Numerator}{Denominator},\ Denominator \neq 0$$

The set of real numbers for which a rational expression is defined is the ________ of the expression. One must exclude all numbers from this set that make the denominator of the rational expression _______.

____________________ Rational Expressions

Common Denominator

____________________ Rational Expressions

No common denominator needed

Multiply straight across

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$$

___________________ Rational Expressions

Multiply by the Reciprocal of the **2nd** Fraction

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}$$

$$\rightarrow \frac{x}{x+3} \qquad Denominator \neq 0$$

$$x+3 \neq 0$$

$$x \neq \_\_\_\_\_\_\_\_$$

$$The\ Rational\ Expression\ is\ undefined\ \ for\ x = \_\_\_\_\_\_\_\_$$

$$\rightarrow \frac{x}{x^2+1} \qquad Denominator \neq 0$$

$$x^2+1 \neq 0$$

$$x \neq \_\_\_\_\_\_\_\_\_ \rightarrow Complex\ numbers$$

$$The\ Rational\ Expression\ is\ defined\ \ for\ all\ \_\_\_\_\_\_\_\_\ \ \_\_\_\_\_\_\_\_\_\_\_\_$$

$$\rightarrow \frac{x-2}{x^2+5x+6} \qquad Denominator \neq 0$$

$$x^2+5x+6 \neq 0$$

$$x \neq \_\_\_\_\_\_\ and\ x \neq \_\_\_\_\_\_\_\_$$

$$The\ Rational\ Expression\ is\ undefined\ \ for\ \_\_\_\_\_, \_\_\_\_\_\_$$

Simplify and find any domain exclusions

$$\frac{5x-10}{x^2-4x+4}=\frac{5(\qquad)}{(\qquad)(\qquad)}$$

$$Denominator \neq 0$$

$$Simplified \rightarrow \frac{5}{x-2}$$

$$(x-2)(x-2)\neq 0$$

$$x \neq \_\_\_\_\_ \; restriction$$

Multiply, simplify, domain restrictions

(Determined _______________ Simplifying)

$$\frac{x^2-9}{x^2}\cdot\frac{x^2-3x}{x^2+6x-27}=\frac{(x+3)(x-3)x(x-3)}{x(x)(x+9)(x-3)}$$

$$Denominator \neq 0$$

$$x(x)(x+9)(x-3)\neq 0$$

$$restrictions: x \neq \_\_\_\_\_, x \neq \_\_\_\_\_, x \neq \_\_\_\_\_$$

$$\frac{(x+3)(x-3)\cancel{x}\,\cancel{(x-3)}}{x(\cancel{x})(x+9)\,\cancel{(x-3)}}=\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$\rightarrow \frac{x^2-25}{3x-3} \div \frac{x^2+10x+25}{x^2+4x-5} = \frac{(x+5)(x-5)}{3(x-1)} \div \frac{(x+5)(x+5)}{(x+5)(x-1)}$$

Denominator $\neq 0$

$$= \frac{\cancel{(x+5)}(x-5)}{3\cancel{(x-1)}} \cdot \frac{\cancel{(x+5)}\cancel{(x-1)}}{\cancel{(x+5)}\cancel{(x+5)}}$$

$= \_\_\_\_\_\_\_\_\_\_$ *simplified*

$x \neq$ ______, ______ *Domain restrictions*

$$\rightarrow \frac{x^2+4x}{x^2+x-6} - \frac{x^2-12}{x^2+x-6} \Rightarrow \textit{ALREADY HAS} \_\_\_\_\_\_\_\_\_\_ \ \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$= \frac{\cancel{x^2}+4x\cancel{-x^2}+12}{x^2+x-6} = \_\_\_\_\_\_\_\_\_\_\_\_ = \frac{4\cancel{(x+3)}}{\cancel{(x+3)}(x-2)}$$

Denominator $\neq 0$

$= \_\_\_\_\_\_\_\_\_\_$ *simplified*

$x \neq$ _____ *and* $x \neq$ _____

$$\rightarrow \frac{7}{9x+4}+\frac{9x}{81x^2-16} \ldots .NEED\ A \_\_\_\_\_\_\_\_\_\_\ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$=\frac{7}{9x+4}\left(\frac{9x-4}{9x-4}\right)+\frac{9x}{(9x+4)(9x-4)}$$

$$=\frac{7(9x+4)+9x}{(9x+4)(9x-4)}=\frac{}{}$$

$$=\frac{72x-28}{(9x+4)(9x-4)}=\frac{}{}\ simplified$$

$$Denominator \neq 0$$

$$(9x+4)(9x-4) \neq 0$$

$$x \neq \_\_\_\_\_\_\_\_\_\_\ and\ x \neq \_\_\_\_\_\_\_\_\_\_\_\_\ restrictions$$

Solving Rational Equations

An equation that is true for **ALL** values of “x” is called an **Identity**

$$3x+2=2x+x+3-1$$
$$3x+2=3x+2$$

$$\underset{IDENTITY}{2=2} \Rightarrow True\,Statement$$

An equation that is **NOT TRUE** for **ANY** value of “x” is called **Inconsistent**

$$2x=2x+3$$

$$\underset{INCONSISTENT}{0=3} \quad \Rightarrow False\,Statement$$

An equation that is only true for **CERTAIN** values of “x” is called a **Conditional Equation**

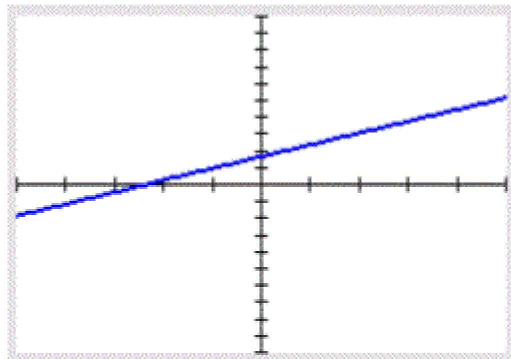
$$3x=2x+4$$

$$\underset{CONDITIONAL\ \ EQUATION}{x=4}$$

Solve

$$\frac{x+6}{5}=2-\frac{x+5}{2}$$

$$Graph\ \frac{x+6}{5}-2+\frac{x+5}{2}=0$$

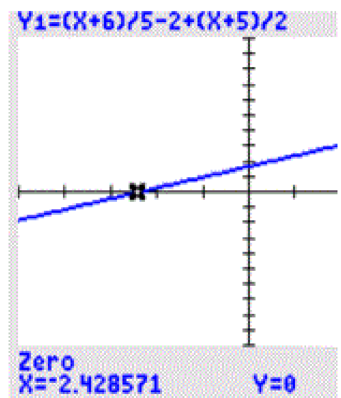


$$NO\ RESTRICTIONS$$

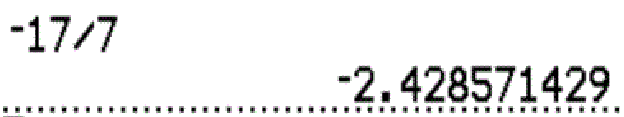
$$Denominator\neq 0$$

$$10\left(\frac{x+6}{5}\right)=10(2)-10\left(\frac{x+5}{2}\right)$$

$$2(x+6)=20-5(x+5)$$

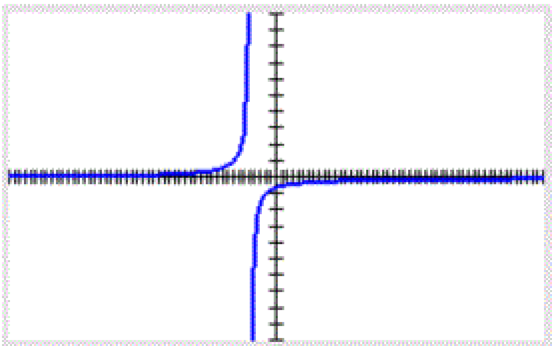


$$x=\underline{\hspace{3cm}}$$



$$\rightarrow \frac{1}{x-2} - \frac{4}{x+5} = \frac{7}{x^2+3x-10}$$

$$Graph: \frac{1}{x-2} - \frac{4}{x+5} - \frac{7}{x^2+3x-10} = 0$$



-7	1.5
-6	3
-5	ERROR
-4	-3
-3	-1.5
-2	-1
-1	-.75
0	-.6
1	-.5
2	ERROR

Restrictions: $x \neq -5, 2$

$$(x+5)\cancel{(x-2)}\left(\frac{1}{\cancel{x-2}}\right) - \cancel{(x+5)}(x-2)\left(\frac{4}{\cancel{x+5}}\right) = \cancel{(x+5)}\,\cancel{(x-2)}\left(\frac{7}{\cancel{(x+5)}\,\cancel{(x-2)}}\right)$$

$$x+5-4(x-2)=7$$

$$x = 2 .... Potential\ Solution$$

$$restrictions: x \neq -5, 2$$

Math102 College Algebra **Unit 3 Outcome/Homework 2**

Students will be able to identify vertical and horizontal asymptotes and graph rational functions. (E-text Section 3.5)

Vertical Asymptote

Horizontal Asymptote

Rules for Finding Horizontal Asymptotes

1. $\dfrac{4x^5+2x^2+1}{-3x^3+7}$ 2. $\dfrac{2x^2+4x^3}{10x^9+7}$ 3. $\dfrac{4x^5+9x^2}{2x^5-7}$

1. $^\circ N > {}^\circ D \Rightarrow$ *No horizontal asymptote*

2. $^\circ N < {}^\circ D \Rightarrow$ *horizontal asymptote is* $y = 0$

3. $^\circ N = {}^\circ D \Rightarrow$ *horizontal asymptote is the ratio of the leading coefficients* $y = \dfrac{n}{d}$

Vertical Asymptote

Where does the denominator = 0? X=_______

$$\rightarrow Domain: h(x) = \frac{x+6}{x^2-36} = \frac{x+6}{(x+6)(x-6)}$$

$$Denominator \neq 0$$

$$(x+6)(x-6) \neq 0$$

$$x \neq \_\_\_\_, x \neq \_\_\_\_\_$$

$$\Rightarrow Vertical\ Asymptotes$$

$$x = \_\_\_\_\_\_\_, x =$$

$$\Rightarrow Horizontal\ Asymptotes$$

$$\frac{^\circ N = 1}{^\circ D = 2} \rightarrow {^\circ N} < {^\circ D}$$

$$\rightarrow y = 0$$

$$\rightarrow Domain\ Vertical\ and\ Horizontal\ Asymptotes$$

$$r(x) = \frac{16x^2 + x}{x^2+5}$$

$$Denominator \neq 0$$

$$x^2 + 5 \neq 0$$

$$x \neq \_\_\_\_\_ \Rightarrow complex\ number$$

$$\Rightarrow no\ restrictions \Rightarrow no\ vertical\ asymptote$$

$$Horizontal\ Asymptote$$

$$\frac{^\circ N = 2}{^\circ D = 2} \Rightarrow {^\circ N} = {^\circ D} \therefore y = \_\_\_\_\_$$

$\rightarrow$ *Domain horizontal and vertical holes*

$$v(x) = \frac{x}{x(x-8)} \qquad Denominator \neq 0$$

$$x(x-8) \neq 0$$

$$x \neq \_\_\_\_, x \neq \_\_\_\_ \quad restrictions$$

Vertical asymptote

$$\frac{\cancel{x}}{\cancel{x}(x-8)} = \frac{1}{x-8} \qquad hole\ at\ x = \_\_\_\_\_$$

$$vertical\ asymptote\ at\ x = \_\_\_\_\_\_$$

Horizontal Asymptote

$$\frac{x}{x(x-8)} = \frac{x^1}{x^2-8x} \Rightarrow \frac{{}^\circ N =}{{}^\circ D =} \Rightarrow {}^\circ N < {}^\circ D \therefore y = \_\_\_\_\_\_\_\_$$

Parent Graph

$y = \frac{1}{x}$

$y = \frac{1}{x^2}$

$\rightarrow Graph\ g(x) = \frac{1}{x-6} + 7$

$Parent\ Graph: y = \frac{1}{x}$

$Two\ Transformations$

$+7 \Rightarrow$

$-6 \Rightarrow$

A company is planning to manufacture mountain bikes. The fixed monthly cost will be \$100,000 and it will cost \$300 to produce each bicycle.

A. Write the cost function, C, of producing x mountain bikes per month.

A. Write the cost function, C, of producing x mountain bikes per month.

$C(x) = 100000 + 300x$

B. Write the average cost function, $\overline{C}$, of producing x mountain bikes per month.

$\overline{C}(x) = \square$

$$\overline{C}(x) = \frac{100000 + 300x}{x}$$

C. Find and interpret $\overline{C}(500)$, $\overline{C}(1000)$, $\overline{C}(2000)$, and $\overline{C}(4000)$.

$\overline{C}(500) = \square$

```
(100000+300*500)/500
                         500
```

Interpret $\overline{C}(500)$.

When $\square$ bicycles are produced in a month, it costs \$ $\square$ to produce each bicycle.

Math102 College Algebra Unit 3 Outcome/Homework 3

Students will be able to find inverse functions on examinations, quizzes, and homework problems. (E-Text section 2.7)

Testing for a Function

Vertical Line Test

Testing for an Inverse

Horizontal Line Test

Function and its inverse

$\rightarrow f(x) \Rightarrow \text{function}$

$f^{-1}(x) \Rightarrow$ *notation that means "the inverse of the function* $f(x)$"

$\rightarrow$ *if a function has an inverse it is called one − to − one*

$\rightarrow$ *for* $f(x)$ *and* $f^{-1}(x)$, *the domain and range swap*

the domain of $f(x)$ *becomes the range of* $f^{-1}(x)$

the range of $f(x)$ *becomes the domain of* $f^{-1}(x)$

$f(x) = \{(-3,-1),(2,7),(3,6),(4,9)\}$

$\text{Domain of } f(x) = \{-3,2,3,4\}$

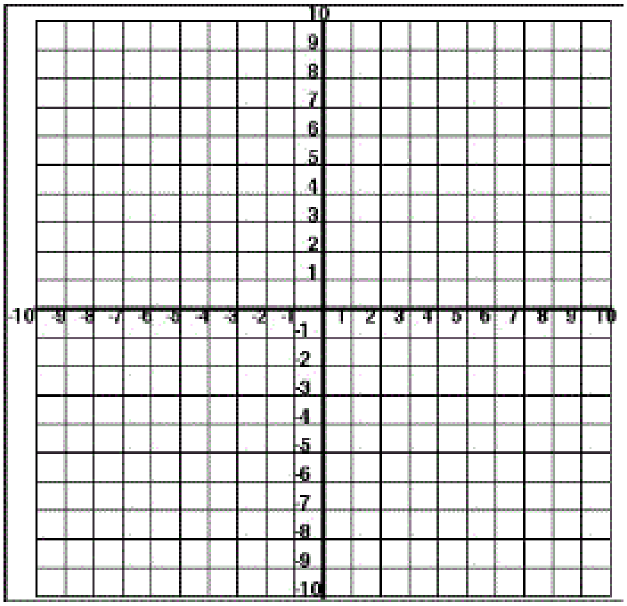
$\text{Range of } f(x) = \{-1,7,6,9\}$

$f^{-1}(x) = \{(-1,-3),(7,2),(6,3),(9,4)\}$

$\text{Domain of } f^{-1}(x) = \{-1,7,6,9\}$

$\text{Range of } f^{-1}(x) = \{-3,2,3,4\}$

The graph of a function and its inverse the graphs reflect across the y=x line.



$$f(x) = 2x - 1$$

$$(0,-1),(2,3),(3,5),(-1,-3)$$

$$f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$$

$$(-1,0),(3,2),(5,3),(-3,-1)$$

How to verify functions are inverses of each other

$$f(x) \qquad\qquad f^{-1}(x) = g(x)$$

$$(f \circ g)(x) = x \Rightarrow f\left[g(x)\right] = x$$

$$(g \circ f)(x) = x \Rightarrow g\left[f(x)\right] = x$$

How to find an inverse

1) Replace f(x) with ________

2) Swap the ______ and ______

3) Solve for ______

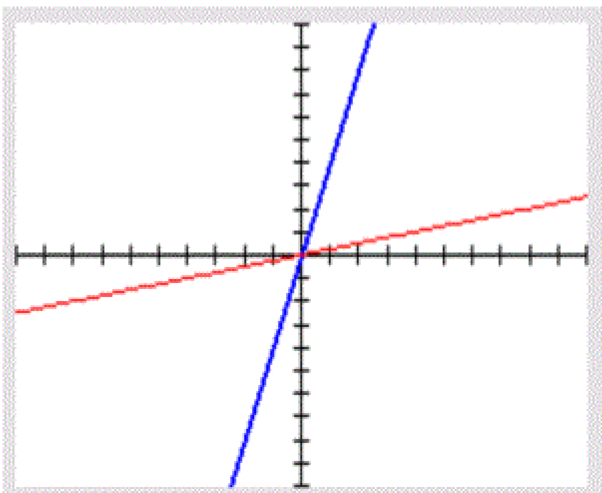
4) Replace _____ with ________

$Determine\ if\ f(x) = 4x\ and\ g(x) = \frac{x}{4}\ are\ inverses\ of\ each\ other$

$f[g(x)] = f\left[\frac{x}{4}\right] =$

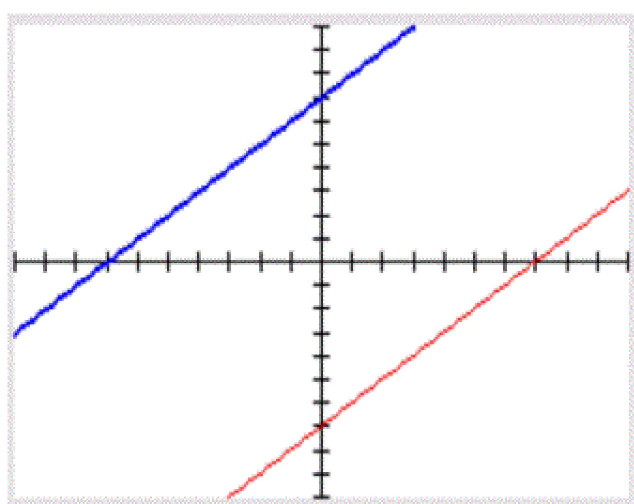
$g[f(x)] = g[4x] =$

$\therefore$ ______ and _______ $are\ inverses\ of\ each\ other$



$the\ function\ f(x) = x + 7\ is\ one - to - one.\ find\ the\ inverse$

1) $f(x) = x + 7$



2)

3)

4)

$$Find\ the\ inverse\ of\ the\ one-to-one\ function\ f(x)=\frac{8x+4}{x-1}$$

1) $f(x)=\dfrac{8x+4}{x-1}$

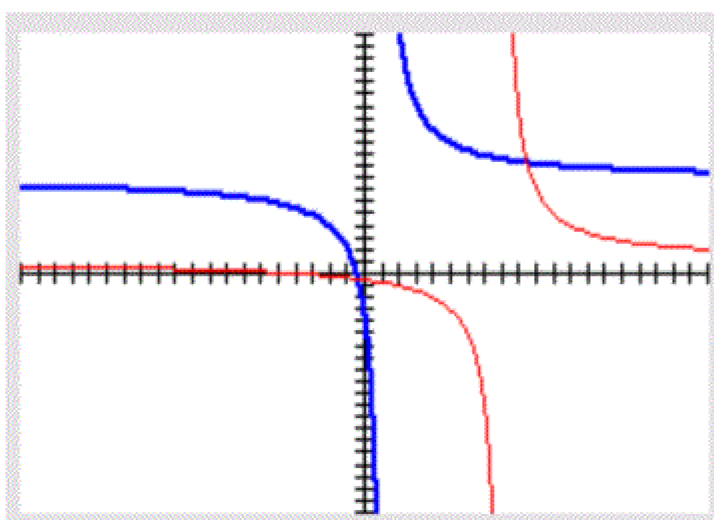
2)

3)

$$xy-x=8y+4$$

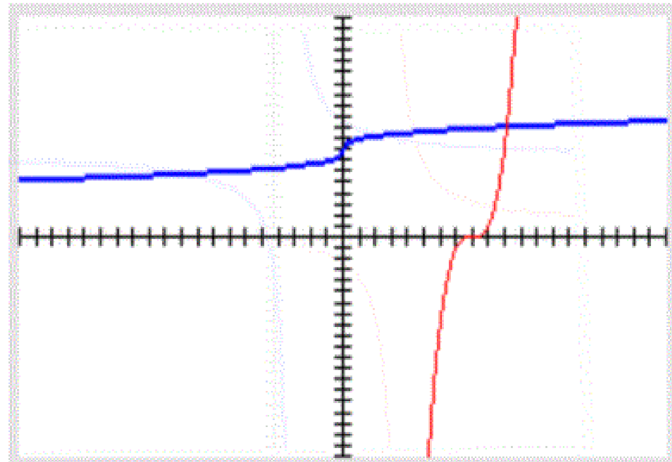
$y=$

4) $f^{-1}(x)=$



Function $f(x) = \sqrt[3]{x} + 8$

a) find $f^{-1}(x)$



1)

2)

3)

4)

b) Domain of $f(x) =$ range of $f^{-1}(x) =$

Range of $f(x) =$ domain of $f^{-1}(x) =$

Let the functions f and g be given by the equations on the right. Evaluate the indicated function without finding an equation for the function.

$f(x) = 5x - 1$

$g(x) = 8x - 1$

$(f \circ g)(0)$

Let the function f be given by the equation $f(x) = 5x - 2$. Evaluate $f^{-1}(3)$ without finding an equation for the function $f^{-1}(x)$.

$f^{-1}(3) =$ ☐

Let the functions f, g, and h be defined by the equations on the right. Evaluate the indicated function without finding an equation for the function.

$g(f[h(0)])$

$$f(x) = 4x - 1$$
$$g(x) = 2x - 1$$
$$h(x) = x^2 + 3x + 4$$

$g(f[h(0)]) =$ ☐

Math102 College Algebra **Unit 3 Outcome/Homework 4**

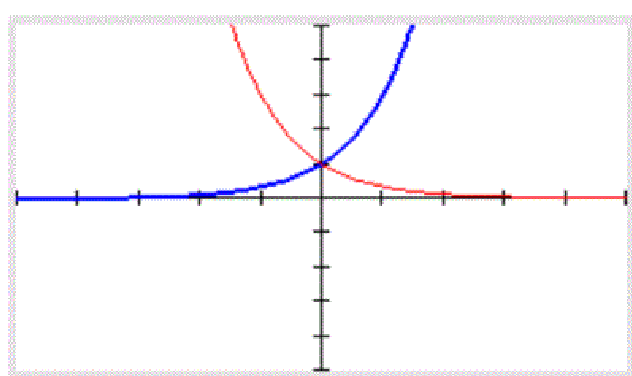
Students will be able to know the basic properties of exponential functions and graph exponential functions on examinations, quizzes, and homework problems. (E-text Section 4.1)

Polynomial Function: $x^3 \ or \ 5x^4 + 3x^2 + 2x + 1$

Exponential Function: $3^x \ or \ 3^{2x+3}$

Graph $f(x) = 3^x$ (Growth)

Graph $f(x) = 3^{-x} = \dfrac{1}{3^x} = \left(\dfrac{1}{3}\right)^x$ (Decay)



$Domain:$

$Range:$

Important Base

$e \rightarrow$ *the natural number*

$e \approx$

Graph $\quad y = e^{x}$ $\qquad\qquad\qquad$ $y = e^{-x}$

$Transformations : Parent : y = 3^{x}$

$\rightarrow y = 3^{x} + 1$.......*Vertical* ______________________

$\rightarrow y = 3^{x} - 5$.......*Vertical* ______________________

$\rightarrow y = 3^{-x}$.....*reflection* ______________________

$\rightarrow y = -3^{x}$.....*reflection* ______________________

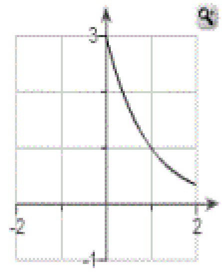
$\rightarrow y = 3^{x+1}$.....*horizontal* ______________________

$\rightarrow y = 3^{x-4}$.....*horizontal* ______________________

what about $\underbrace{y = 1^{x} \quad or \quad y = 1^{-x}}_{BOTH\ HORIZONTAL\ LINES}$

Match the graph to one of the following functions.

$y=3^{x}$	$y=3^{-x}$
$y=-3^{x}$	$y=-3^{-x}$
$y=3^{x}-1$	$y=3^{x-1}$
$y=3^{1-x}$	$y=1-3^{x}$



Which function is represented by the graph?

A. $y=1-3^{x}$

B. $y=3^{x-1}$

C. $y=3^{x}-1$

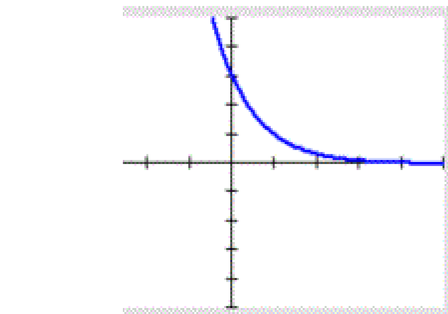
D. $y=3^{x}$

E. $y=3^{1-x}$

F. $y=3^{-x}$

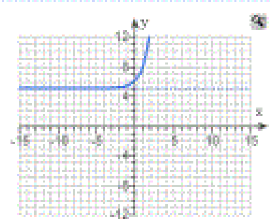
G. $y=-3^{-x}$

H. $y=-3^{x}$



Use transformations of the graph of $f(x)=e^{x}$ to graph the given function. Be sure to give the equation of the asymptote. Use the graph to determine the function's domain and range.

$g(x)=e^{x}+5$



Graph $g(x)=e^{x}+5$. Use the graphing tool to graph the function.

What equation represents the asymptote of $g(x)=e^{x}+5$?

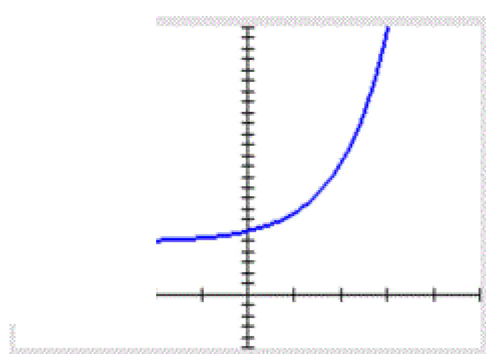
$y=5$ (Type an equation.)

What is the domain of $g(x)=e^{x}+5$?

$(-\infty,\infty)$
(Type your answer in interval notation.)

What is the range of $g(x)=e^{x}+5$?

$(5,\infty)$
(Type your answer in interval notation.)



$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

A (Accumulated Amount) = balance in the account when **P (principal)** dollars are invested at an **annual interest "r"** for **"t" years** where **"n" is the number of compounding periods.**

Daily….n=______ annually…..n=______

Semiannually…..n=______ monthly…..n=_____

Quarterly……n=______

Continuous …. Switch to a different formula!

Continuously Compounding Interest

$$A = Pe^{rt}$$

Use the compound interest formulas $A = P\left(1+\frac{r}{n}\right)^{nt}$ and $A = Pe^{rt}$ to solve the problem given. Round answers to the nearest cent.

Find the accumulated value of an investment of $15,000 for 4 years at an interest rate of 5% if the money is **a.** compounded semiannually; **b.** compounded quarterly; **c.** compounded monthly **d.** compounded continuously.

a. What is the accumulated value if the money is compounded semiannually?

$☐ (Round your answer to the nearest cent.)

```
15000(1+.05/2)^(2*4)
                18276.04346
```

b. Compounded Quarterly (n=4)

```
15000(1+.05/4)^(4*4)
                18298.34322
```

d. Compounded Continuously

```
15000*e^(.05*4)
                18321.04137
```

Suppose you have $13,000 to invest. Which of the two rates would yield the larger amount in 3 years: 8% compounded daily or 7.88% compounded continuously?

Which of the two rates would yield the larger amount in 3 years?

- ○ 8% compounded daily
- ○ 7.88% compounded continuously

$8\%\ compounded\ daily\ (n = \_\_\_\_\_\_\_)$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$A =$

```
13000(1+.08/365)^(365*3)
                16525.80436
```

$7.88\%\ compounded\ continuously$

$$A = Pe^{rt}$$

$A =$

```
13000e^(.0788*3)
                16466.85146
```

The 7.88% account, compounded continuously yields the __________ amount.

The 1982 explosion at a nuclear lab sent about 1000 kilograms of a radioactive element into the atmosphere. The function $f(x) = 1000(0.5)^{\frac{x}{30}}$ describes the amount, f(x), in kilograms, of a radioactive element remaining in the area x years after 1982. If even 100 kilograms of the radioactive element remains in the atmosphere, the area is considered unsafe for human habitation. Find f(40) and determine if the area will be safe for human habitation by 2022.

$f(40) \approx$ ☐ (Type an integer or a decimal rounded to the nearest tenth as needed.)

```
1000*.5^(40/30)
                396.850263
```

Math102 College Algebra **Unit 3 Outcome/Homework 5**

Students will be know the basic properties of logarithmic functions and graph logarithmic functions on examinations, quizzes, and homework problems. (E-text Section 4.2)

Logarithmic Functions

$$y = \log_b(x) \Rightarrow b^y = x$$

$$\log_b(x) = y \quad \Rightarrow Log\ Form$$

$$b^y = x \quad \Rightarrow Exponential$$

$$y = \log_{10}(x) = \log(x) \Rightarrow Common\ Log$$

$$y = \log_e(x) \quad \Rightarrow the\ natural\ \log$$

$$y = \ln(x)$$

Log Graph

You cannot take the log of a negative number!

Domain:

Range:

Properties of Logs

$\rightarrow \log_b(b) = 1 \qquad\qquad b^{?} = b$

$\rightarrow \log_b(1) = 0 \qquad\qquad b^{?} = 1$

$\rightarrow \log_b(b^x) = x \qquad\qquad b^{?} = b$

$\log_b(b^x)$

$x\,\overbrace{\cancel{\log_b(b)}}^{1} = x$

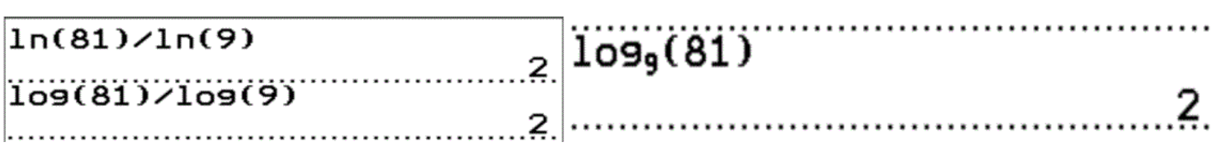
$\rightarrow b^{\log_b(x)} = x$

Log Conversion

$$\log_b(a) = \frac{\log(a)}{\log(b)} = \frac{\ln(a)}{\ln(b)}$$

$\log_9(81) = 2 \qquad\qquad \square^{2} = \underline{\qquad}$

$$\log_9(81) = \frac{\square}{\square} = \frac{\square}{\square}$$



Transformations

$g(x) = \boxed{9} + \log_2(x) \rightarrow$ *Vertical shift* ______________________

$g(x) = \log_5(x+9) - 7 \rightarrow$ "-7" *Vertical shift* __________________

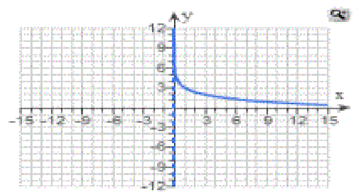
$\rightarrow$ "$+9$" *Horizontal shift* _______________

$g(x) = -\ln(x) \rightarrow$ "-1" *reflection across* ________________

Begin by graphing f(x) = **ln** x. Use transformations of this graph to graph the given function. Graph and give the equation of the asymptote. Use the graphs to determine the function's domain and range.

g(x) = 3 − **ln** x

Graph g(x) = 3 − **ln** x. Graph the asymptote of g(x) as a dashed line. Use the graphing tool to graph the function.



What is the equation of the vertical asymptote of g(x)?

x = 0

What is the domain of g(x) = 3 − **ln** x?

(0,∞)

(Simplify your answer. Type your answer in interval notation.)

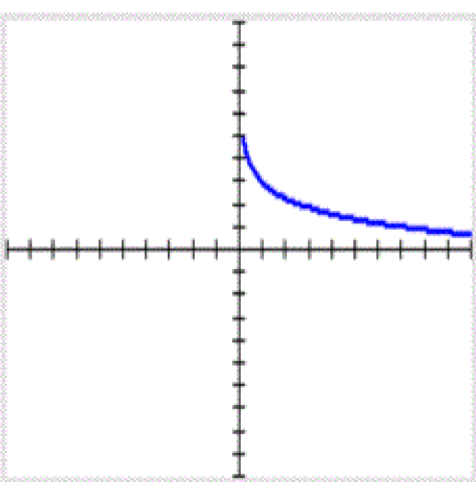
What is the range of g(x) = 3 − **ln** x?

(−∞,∞)

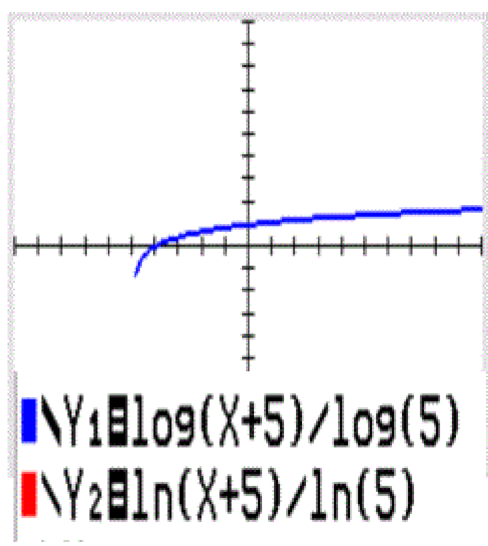
(Simplify your answer. Type your answer in interval notation.)

Graph

$3 - \ln(x)$



$g(x) = \log_5(x+5)$



$$\rightarrow 4 = \log_7(2401) \qquad\qquad \square^4 = \boxed{}$$

$$\rightarrow 5 = \log_3(m) \qquad\qquad \square^5 = m$$

$$\rightarrow \log_4(16) = y \qquad\qquad \square^y = \square$$

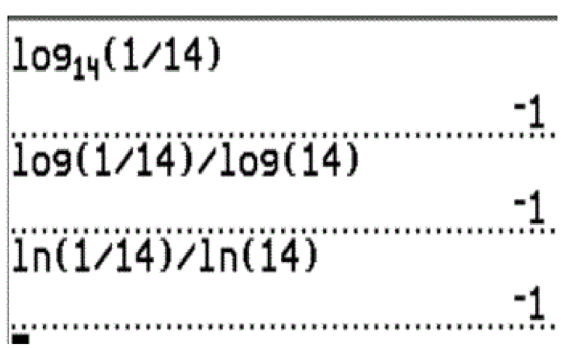
$$\rightarrow 8^3 = 512 \qquad\qquad \log_{\square}\boxed{(\quad)} = 3$$

$$\rightarrow \frac{1}{125} = 5^{-3} \qquad\qquad \log_{\square}\boxed{\frac{}{}} = \square$$

$$\rightarrow \sqrt{25} = 5$$

$$25^{\frac{1}{2}} = 5 \qquad\qquad \log_{\square}\square = \frac{1}{2}$$

$$\rightarrow \log_{14}\left(\frac{1}{14}\right) = -1$$



$\rightarrow \log_3 \sqrt{3}$

$\log_3 3^{\frac{1}{2}}$

$\frac{1}{2}\log_3(3)$

$\frac{1}{2}(1) = \frac{1}{2}$

$\Rightarrow \log_3 \sqrt{3} = \frac{1}{2}$

```
log₃(√3)
                              .5
Ans▸Frac
                              1/2
log(√3)/log(3)
                              .5
Ans▸Frac
                              1/2
```

$\rightarrow \log\left(\frac{1}{100}\right) \quad \Rightarrow \log_{10}\left(\frac{1}{100}\right) = y$

$\square^{y} = \frac{\square}{\square}$

```
log(1/100)
                              -2
```

$y =$ ______

$\rightarrow \ln\left(\frac{1}{e}\right) \quad \Rightarrow \quad \log_{\square}\left(\frac{\square}{\square}\right) = y$

$$\square^{y} = \frac{\square}{\square}$$

$$\square^{y} = e^{\square}$$

$$y = \_\_\_\_\_\_$$

```
ln(1/e)
                                   -1
```

The percentage of adult height attained by girls who are x years old can be modeled by

$f(x) = 62 + 35\,\mathbf{log}\,(x - 4)$

where x represents the girl's age (from 5 to 15) and f(x) represents the percentage of her adult height. Use this function to determine approximately what percent of her adult height girls are at age 6.

The approximate percent of her adult height is ☐%. (Round to two decimal places.)

```
62+35log(6-4)
                  72.53604985
```

The loudness level of a sound, D, in decibels, is given by the formula $D = 10 \log (10^{12} I)$, where I is the intensity of the sound, in watts per meter^2. Decibel levels range from 0, a barely audible sound, to 160, a sound resulting in a ruptured eardrum. The sound of a certain animal can be heard 500 miles away, reaching an intensity of 6.2×10^7 watts per meter^2. Determine the decibel level of this sound. At close range, can the sound of this animal rupture the human eardrum?

The decibel level of this animal's sound is approximately ☐ decibels.
(Round to the nearest whole number as needed.)

$10\log(10^{12} * 6.2 * 10^7)$

197.9239169

Math102 College Algebra **Unit 3 Outcome/Homework 6**

Students will be able to solve exponential and logarithmic equations using the rules of logarithms and exponents on examinations, quizzes, and homework problems. (E-text Section 4.3, 4.4)

Properties of Logs

$$\rightarrow \log_b(MN) = \log_b(M) + \log_b(N)$$

$$\log_4(2 \cdot 3) = \log_4(2) + \log_4(3)$$

```
log4(2*3)
                    1.29248125
log4(2)+log4(3)
                    1.29248125
```

$$\log_4(2 \cdot 3) = \log_4(2) + \log_4(3)$$

$$\textit{using } \log \textit{ conversion}$$

$$\frac{\log(2 \cdot 3)}{\log(4)} = \frac{\log(2)}{\log(4)} + \frac{\log(3)}{\log(4)}$$

```
log(2*3)/log(4)
                    1.29248125
log(2)/log(4)+log(3)/log(▸
                    1.29248125
```

$$\rightarrow \log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$$

$$\rightarrow \log_b M^p = p \log_b(M)$$

Change of Base

$$\log_b(M) = \frac{\log_a(M)}{\log_a(b)} = \frac{\log_{10}(M)}{\log_{10}(b)} = \frac{\log(M)}{\log(b)} = \frac{\ln(M)}{\ln(b)}$$

$$\rightarrow \log_2(4x) = \log_2(4) + \log_2(x)$$

$$= 2 + \log_2(x)$$

$$\rightarrow \log_t x^3 \quad = \quad \boxed{}\log_t(x)$$

$$\rightarrow \log_3\left(\frac{27}{y}\right) = \log_{\Box}\boxed{(\quad)} - \log_{\Box}\boxed{(\quad)}$$

$$= \boxed{} - \log_3\boxed{}$$

$$= \frac{\boxed{\log(\quad)}}{\boxed{\log(\quad)}} = \boxed{}$$

$$\rightarrow \log_4\left(\frac{64}{\sqrt{x+8}}\right) = \log_{\Box}\boxed{(\quad)} - \log_{\Box}\boxed{\left(\sqrt{\quad}\right)}$$

$$= \log_{\Box}\boxed{^{\Box}} - \log_{\Box}\boxed{(\quad)^{\frac{\Box}{\Box}}}$$

$$= \boxed{}\log_{\Box}\boxed{} - \boxed{\frac{}{}}\log_{\Box}\boxed{(\quad)}$$

$$=$$

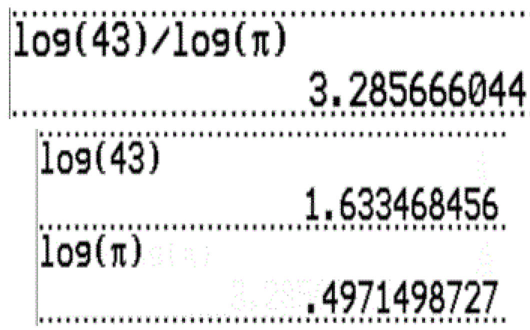
$\rightarrow \log(4x+1)-\log(x)=\log\left(\frac{\square}{\square}\right)$

$\rightarrow 2\ln(x)-\frac{1}{7}\ln(y)$

$=\ln(x)^{\square}-\ln(y)^{\square}$

$=\ln\left(\frac{\square}{\square}\right)$

$\rightarrow \log_{\pi}(43)=\frac{\log\square}{\log\square}$



$\rightarrow b^{M}=b^{N}$

$\ln(b^{M})=\ln(b^{N})$

$=\square\ln(b)=\square\ln(b)$

$\rightarrow \square=\square$

Solve for x

$\rightarrow x\ln(9)=\ln(28)$

$x=\frac{\square}{\square}$

$$\rightarrow e^{0.5x} = 5$$

$$= \ln e^{0.5x} = \ln(5)$$

$$= \square \ln(e) = \ln(5) \qquad Note: \ln(e) = 1$$

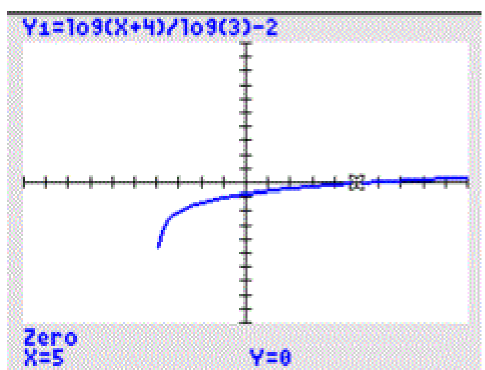
$$= \square = \ln(5)$$

$$x = \frac{\square}{\square}$$

$$\rightarrow \log_3(x+4) = 2 \qquad Graph \log_3(x+4) - 2 = 0$$

$$= \square^2 = \square$$

$$x = \square$$



-3	-2
-2	-1.369
-1	-1
0	-.7381
1	-.535
2	-.3691
3	-.2288
4	-.1072
5	0
6	.0959
7	.18266

$$\log_2(x)+\log_2(x+6)=2$$

$$\log_2\left(\square\left(\square\right)\right)=2$$

$$\log_2\left(\square\right)=2$$

$$\square^2=\square$$

$$x^2+6x=4$$

$$x^2+6x-4=0$$

$$a=\square \quad b=\square \quad c=\square$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

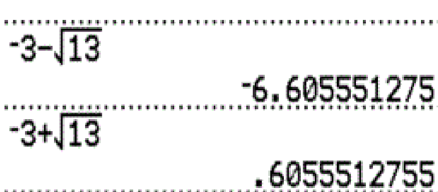
$$x=\square\,,\,\square$$

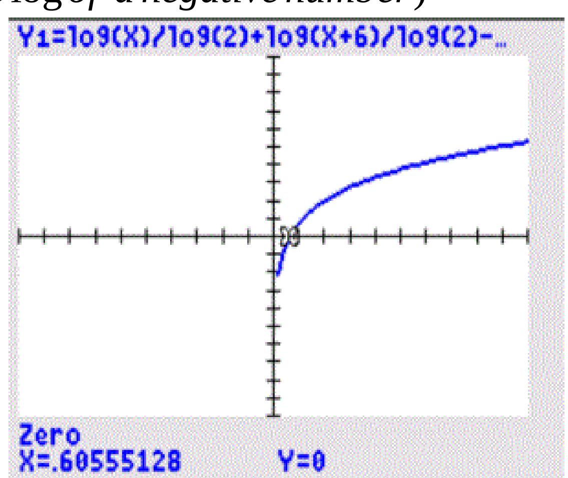
$NOT\ A\ SOLUTION$ ($Cannot\ take\ the$ log $of\ a\ negative\ number$)

$$\log_2(x)+\log_2(x+6)=2$$

$$\log_2(x)+\log_2(x+6)-2=0$$

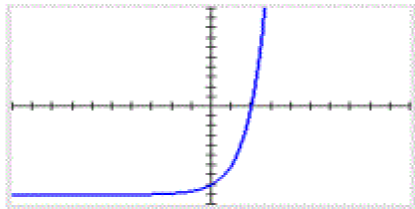
$$\frac{\log(x)}{\log(2)}+\frac{\log(x+6)}{\log(2)}-2=0$$





$\rightarrow 3^{x} = 9$

$x = 2$



$\ln(3^{x}) = \ln(9)$

$x\ln(3) = \ln(9)$

$$x = \frac{\ln(9)}{\ln(3)} = 2$$

$\rightarrow 7^{2x-1} = 343$

$7^{2x-1} = 7^{\Box}$

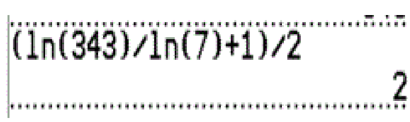
$\Box = \Box$

$x = \Box$

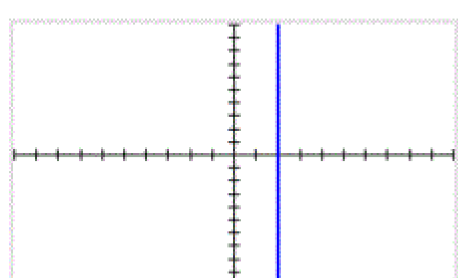
$\ln(7^{2x-1}) = \ln 343$

$(2x-1)\ln(7) = \ln(343)$

$x = \Box$

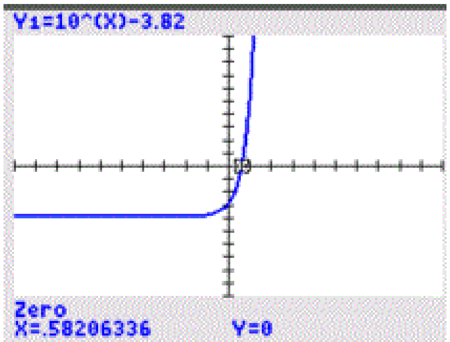


$Graph\, 7^{2x-1} - 343 = 0$



$$10^x = 3.82$$

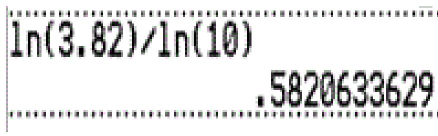
$$Graph\,10^x - 3.82 = 0$$



$$\log 10^x = \log(3.82)$$

$$\boxed{\quad}\log(10) = \log(3.82)$$

$$x = \boxed{\frac{\quad}{\quad}} = \boxed{\frac{\quad}{\quad}}$$



$$\rightarrow e^{4x-3} - 7 = 23108$$

$$e^{4x-3} = \boxed{\quad\quad}$$

$$\ln e^{4x-3} = \ln(23115)$$

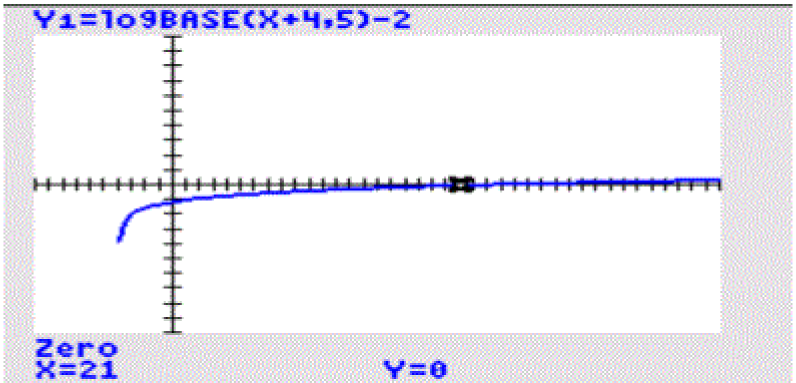
$$\boxed{\quad\quad}\ln e = \ln(23115)$$

$$\boxed{\quad\quad} = \ln(23115)$$

$$x = \boxed{\frac{\quad\quad\quad}{\quad\quad\quad}}$$

$\rightarrow \log_5(x+4)=2$

$\log_5(x+4)-2=0$ *or* $\dfrac{\log(x+4)}{\log(5)}-2=0$



$\log_5(x+4)=2$

$\square^2 = x+4$

$x=\square$

$\rightarrow 9\ln(2x)=45$

$9\ln\left[2\left(\dfrac{e^5}{2}\right)\right]=45$

$\ln(2x)=\square$

$\ln_e(2x)=\square$

$\square^{\square}=\square$

$9\ln e^5=45$

$\square(9)\ln(e)=45$

$\square=\square$

$x=\dfrac{\square}{\square}$

Math102 College Algebra **Unit 3 Outcome/Homework 7**

Students will be able to solve application problems involving exponential and logarithmic functions using the rules of logarithms and exponents on examinations, quizzes, and homework problems. (Section 4.4)

The formula $A = 24.2\,e^{0.0765t}$ models the population of a US state, A, in millions, t years after 2000.

a. What was the population of the state in 2000?
b. When will the population of the state reach 31.4 million?

a. In 2000, the population of the state was ☐ million.

$$a)\quad A = 24.2e^{0.0765(0)}$$
$$= 24.2e^{0}$$
$$= 24.2$$

$$b)\quad 31.4 = 24.2e^{0.0765(t)}$$

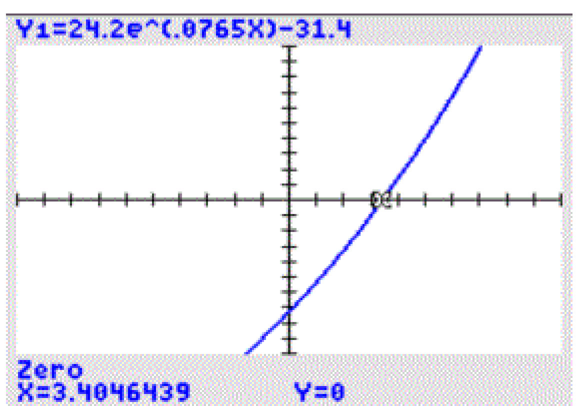
$$\ln e^{0.0765(t)} = \ln \frac{31.4}{24.2}$$

$$0.0765t\ln(e) = \ln \frac{31.4}{24.2}$$

$t =$ ☐ *rounded to the nearest year*

$t =$ ☐

$$Graph \quad 31.4 = 24.2e^{0.0765t}$$
$$\rightarrow 24.2e^{0.0765t} - 31.4 = 0$$



Complete the table for a savings account subject to 2 compoundings yearly.

$$\left[A = P\left(1+\frac{r}{n}\right)^{nt}\right]$$

Amounted Invested	Number of Compounding Periods	Annual Interest Rate	Accumulated Amount	Time t in Years
$11,500	2	6.75%	$25,000	?

Let A represent the accumulated amount, P the amount invested, n the number of compounding periods, r the annual interest rate, and t the time. Find the time, t.

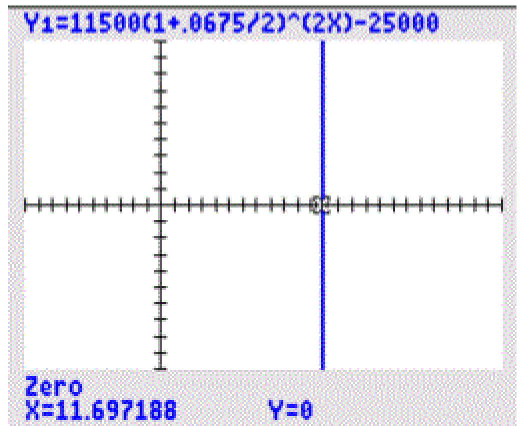
t = ☐ years

(Do not round until the final answer. Then round to one decimal place as needed.)

$$25000 = 11500\left(1+\frac{.0675}{2}\right)^{2(t)}$$

$$Graph\ 11500\left(1+\frac{.0675}{2}\right)^{2(t)} - 25000 = 0$$

11	-1131
12	507.65



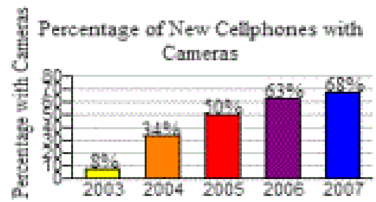
By hand ...

$$25000 = 11500\left(1+\frac{.0675}{2}\right)^{2t}$$

$$t = \boxed{\frac{\quad}{\quad}}\ \textit{in exact form}$$

```
(ln(25000/11500))/(2*ln(1▸
                11.69718846
```

The days when something happens and it is not captured on camera appear to be over. The bar graph shows that by 2007, 68% of cellphones sold in a country were equipped with a camera. The data can be modeled by the function $f(x) = 8 + 43 \ln x$, where $f(x)$ is the percentage of new cellphones with cameras x years after 2002.



a. Use the function to determine the percentage of new cellphones with cameras in 2007. Does this overestimate or underestimate the percent displayed by the graph? By how much?

Determine the percentage of new cellphones with cameras in 2007 using the given function.

☐% (Round to the nearest percent as needed.)

$$f(5) = 8 + 43\ln(5) =$$

Does this overestimate or underestimate the percent displayed by the graph?

- ○ A. Underestimates
- ○ B. Overestimates

By how much?

☐%

b. If trends shown from 2003 through 2007 continue, use the function to determine by which year 86% of new cellphones will be equipped with cameras.

By which year will 86% of new cellphones will be equipped with cameras?

☐ (Round to the nearest year as needed.)

$$f(x) = 8 + 43\ln(x)$$

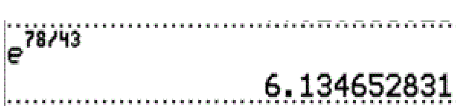
$$86 = 8 + 43\ln(x)$$

$$78 = 43\ln(x)$$

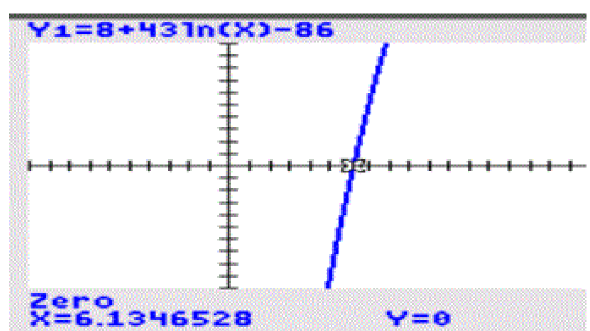
$$\ln(x) = \frac{78}{43}$$

$$\log_e(x) = \frac{78}{43}$$

$$e^{\frac{78}{43}} = x$$



Graph $8 + 4\ln(x) - 86 = 0$



Exponential Growth/Decay

$$A = A_0 e^{kt}$$

A = Amount Remaining

A_0 = Initial Amount

K = growth rate/ decay rate – k>0 = growth and k<0 = decay

t= time in years

t=0 our amount is A_0

t=any other time out amount is A

A bird species in danger of extinction has a population that is decreasing exponentially $(A = A_0 e^{kt})$. Seven years ago the population was at 1600 and today only 1000 of the birds are alive. Once the population drops below 300, the situation will be irreversible. When will this happen?

The population will drop below 300 birds approximately ☐ years from now.

(Do not round until the final answer. Then round up to the nearest whole number as needed.)

$$A = A_0 e^{kt}$$

$$300 = 1000e^{kt}$$

plug in k

find k

$$1000 = 1600e^{k(7)}$$

$$e^{7k} = \frac{1000}{1600}$$

$$e^{7k} = \frac{\square}{\square}$$

$$\ln(e^{7k}) = \ln\frac{5}{8}$$

$$\square \ln(e) = \ln\left(\frac{\square}{\square}\right)$$

$$k = \frac{\square}{\square}$$

Math102 College Algebra **Unit 3 Outcome/Homework 8**

Students will be able to solve a system of two linear equations on examinations, quizzes, and homework problems. (Section 5.1)

Graph

No solution	*Single solution*	*Infinite solutions*
Inconsisten system	*consistent system*	*dependent system*

How to solve Linear Systems

1) Graphing 2) Substitution 3) Elimination

Determine if (1,-2) is a solution to the system

$$y = 4x - 6$$
$$3x + 2y = -1$$

$$y = 4x - 6$$
$$(-2) = 4(1) - 6$$

$$3x + 2y = -1$$
$$3(1) + 2(-2) = -1$$

$\therefore (1,-2)$ *is a solution to the system*

→Determine if (2,-7) is a solution to the system

$$5x+2y=-4$$
$$2x-3y=27$$

$5x+2y=-4$ $\qquad\qquad$ $2x-3y=27$

$\therefore (2,-7)\, is \boxed{}\, a\, solution\, to\, the\, system$

Solve the system

$2x-2y=9$

$y=2x-5$

Substitution

substitute $y=2x-5$ *in* $2x-2\boxed{y}=9$

$2x-2\boxed{(\quad)}=9$

find the $y-$*value by plugging in*

$x=\frac{1}{2}$ *in either equation*

$y=2x-5$

$y=2\left(\frac{}{}\right)-5$

$y=\boxed{}$

$\therefore \boxed{(\quad,\quad)}$ *is a solution to the system*

Elimination

solve the system

$x + 5y = 21$

$4x + 3y = 21$

$\rightarrow \boxed{(\quad)}(x) + \boxed{(\quad)}(5y) = \boxed{(\quad)}(21) \quad \rightarrow \quad \boxed{\quad} - \boxed{\quad} = \boxed{\quad}$

$\quad 4x \quad + \quad 3y \quad = \quad -1 \qquad\qquad\qquad 4x \quad + \quad 3y \quad = \quad -1$

$\rightarrow \boxed{\quad} = \boxed{\quad}$

$y = \boxed{\quad}$

$\therefore \boxed{(\quad , \quad)}$ *is the solution to the system*

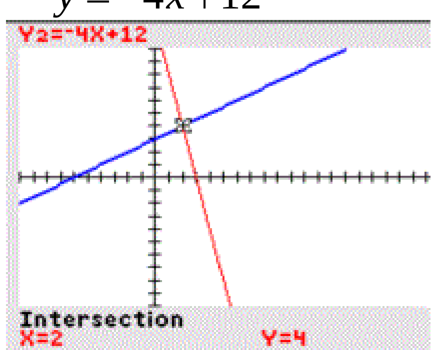
Check!

Graphing $\rightarrow$ *solve both equations for* $\boxed{\quad}$

Solve the system

$x - 2y = -6 \quad \rightarrow \quad y = \boxed{\frac{\quad}{\quad}}x + \boxed{\quad}$

$y = -4x + 12 \qquad\qquad y = -4x + 12$



-2	2	20
-1	2.5	16
0	3	12
1	3.5	8
2	4	4
3	4.5	0
4	5	-4

$Solve\ the\ system$

$5x = y - 3 \quad \rightarrow \quad 5x - y = -3$

$5x - y = 7 \quad\quad\quad\quad 5x - y = 7$

$Elimination / Addition\ Method$

$-(5x) - (-y) = -(-3) \quad \rightarrow \quad \square + \square = \square$

$5x \quad - y \quad = \quad 7 \quad\quad\quad\quad 5x \quad - \quad y \quad = \quad 7$

$\square \neq \square$

$\therefore\ false\ statement \rightarrow \boxed{\quad\quad\quad\quad}$

$parallel\ lines \rightarrow \boxed{\quad\quad\quad\quad}$

$Solve\ the\ system\ by\ substitution$

$x = 6y + 5$

$2x - 12y = 10$

$plug\ \ x = 6y + 5\ \ in\ \ 2\boxed{x} - 12y = 10$

$2\boxed{(\quad\quad)} - 12y = 10$

$\square = \square$

$\therefore true\ statement \rightarrow \boxed{\quad\quad\quad , \quad\quad\quad}$

$\boxed{\quad\quad\quad}$ $number\ of\ solutions$

Solve the system

$$\frac{x}{3}-\frac{y}{3}=-1 \quad \rightarrow \textit{clear the fraction by } \square \rightarrow x-y=-3$$

$$x+2y=-9 \qquad\qquad x+2y=-9$$

$$\square(x-y)=\square(-3) \quad \rightarrow \quad \square-\square=\square$$

$$x + 2y = -9 \qquad\qquad x + 2y = -9$$

Find y by substituting $x=\square$

in either equation

$$x=\square$$

$$\therefore \boxed{(\quad , \quad)}$$

A company that manufactures small canoes has a fixed cost of $24,000. It costs $120 to produce each canoe. The selling price is $240 per canoe. (In solving this exercise, let x represent the number of canoes produced and sold.)

a. Write the cost function.

C(x) = ☐ (Type an expression using x as the variable.)

b. Write the revenue function.

R(x) = ☐ (Type an expression using x as the variable.)

c. Determine the break-even point.

☐ (Type an ordered pair. Do not use commas in large numbers.)

This means that when the company produces and sells the break-even number of canoes

- ○ A. there is more money coming in than going out.
- ○ B. there is less money coming in than going out.
- ○ C. the money coming in equals the money going out.
- ○ D. there is not enough information.

When a crew rows with the current, it travels 22 miles in 2 hours. Against the current, the crew rows 10 miles in 2 hours. Let x = the crew's rowing rate in still water and let y = the rate of the current. Find the rate of rowing in still water and the rate of the current.

The table below summarizes the given information.

	Rate	×	Time	=	Distance
Rowing with current	x + y		2		22
Rowing against current	x − y		2		10

The rate of rowing in still water is $\square$ miles per hour.

The rate of the current is $\square$ miles per hour.

$$Distance = Rate(time) \quad \rightarrow \quad d = rt$$

Solve this system

$$22 = 2(x + y)$$
$$10 = 2(x - y)$$

A rectangular lot whose perimeter is 360 ft is fenced along three sides. An expensive fencing along the lot's length cost $35 per foot. An inexpensive fencing along the two side widths costs only $5 per foot. The total cost of the fencing along the three sides comes to $4550. What are the lot's dimensions?

$$p = 360 = 2l + 2w$$
$$4550 = 35l + 10w$$

$$\rightarrow l + w = 180 \qquad \rightarrow \qquad \square l + \square w = \square (180)$$
$$7l + 2w = 910 \qquad\qquad 7l + 2w = 910$$

$$l = \square \qquad w \square$$